ECL 4340

POWER SYSTEMS

Lecture 17
Symmetrical Components, Unbalanced Fault Analysis

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1

Announcements

- Be reading Chapters 8 and 9
- HW 9 is uploaded, due November 11, Friday.
- Exam II is on November 8, Tuesday.

2

Analysis of Unsymmetric Systems

- Except for the balanced three-phase fault, faults result in an unbalanced system.
- The most common types of faults are single line-ground (SLG) and line-line (LL). Other types are double line-ground (DLG), open conductor, and balanced three phase.
- System is only unbalanced at the point of fault!
- The easiest method to analyze unbalanced system operation due to faults is through the use of symmetrical components

3

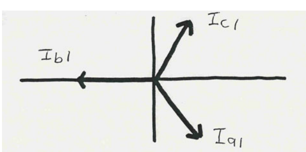
Symmetric Components

- The key idea of symmetrical component analysis is to decompose the system into three sequence networks. The networks are then coupled only at the point of the unbalance (i.e., the fault)
- The three sequence networks are known as the
  - positive sequence (this is the one we've been using)
  - negative sequence
  - zero sequence

4

Positive Sequence Sets

- The positive sequence sets have three phase currents/voltages with equal magnitude, with phase b lagging phase a by 120°, and phase c lagging phase b by 120°.
- We've been studying positive sequence sets

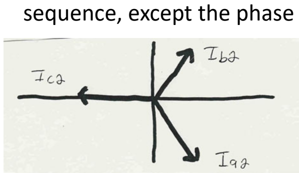


Positive sequence sets have zero neutral current

5

Negative Sequence Sets

- The negative sequence sets have three phase currents/voltages with equal magnitude, with phase *b* leading phase *a* by 120°, and phase *c* leading phase b by 120°.
- Negative sequence sets are similar to positive sequence, except the phase order is reversed

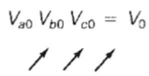


Negative sequence sets have zero neutral current

6

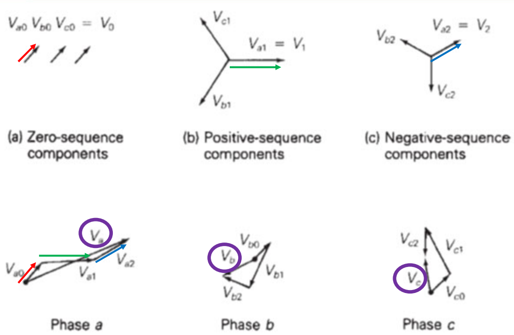
Zero Sequence Sets

- Zero sequence sets have three values with equal magnitude and angle.
- Zero sequence sets have neutral current



7

Zero Sequence Sets



8

Sequence Set Representation

- **Any** arbitrary set of three phasors, say I_a, I_b, I_c can be represented as a sum of the three sequence sets:

$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$I_b = I_b^0 + I_b^+ + I_b^-$$
$$I_c = I_c^0 + I_c^+ + I_c^-$$

where

I_a^0, I_b^0, I_c^0 is the zero sequence set

I_a^+, I_b^+, I_c^+ is the positive sequence set

I_a^-, I_b^-, I_c^- is the negative sequence set

9

Conversion from Sequence to Phase

Only three of the sequence values are unique, I_a^0, I_a^+, I_a^-; the others are determined as follows:

$\alpha = 1\angle 120°$ $\quad \alpha + \alpha^2 + \alpha^3 = 0 \quad \alpha^3 = 1$

$I_a^0 = I_b^0 = I_c^0$ (since by definition they are all equal)

$I_b^+ = \alpha^2 I_a^+ \quad I_c^+ = \alpha I_a^+ \quad I_b^- = \alpha I_a^- \quad I_c^- = \alpha^2 I_a^-$

$$
\begin{aligned}
I_a &= I_a^0 + I_a^+ + I_a^- \\
I_b &= I_b^0 + I_b^+ + I_b^- = I_a^0 + \alpha^2 I_a^+ + \alpha I_a^- \\
I_c &= I_c^0 + I_c^+ + I_c^- = I_a^0 + \alpha I_a^+ + \alpha^2 I_a^-
\end{aligned}
$$

10

Conversion from Sequence to Phase

$$
\begin{aligned}
I_a &= I_a^0 + I_a^+ + I_a^- \\
I_b &= I_b^0 + I_b^+ + I_b^- = I_a^0 + \alpha^2 I_a^+ + \alpha I_a^- \\
I_c &= I_c^0 + I_c^+ + I_c^- = I_a^0 + \alpha I_a^+ + \alpha^2 I_a^-
\end{aligned}
$$

$$
\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \mathrm{I}_a^0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \mathrm{I}_a^+ \begin{bmatrix} 1 \\ \alpha^2 \\ \alpha \end{bmatrix} + I_a^- \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}
$$

11

Conversion Sequence to Phase

Define the symmetrical components transformation matrix

$$
\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}
$$

Then $\mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \mathbf{A} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \mathbf{A} \begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = \mathbf{A}\, \mathbf{I}_s$

12

Conversion Phase to Sequence

By taking the inverse we can convert from the phase values to the sequence values

$$\mathbf{I}_s = \mathbf{A}^{-1}\mathbf{I}$$

$$\text{with} \quad \mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

Sequence sets can be used with voltages as well as with currents

13

Symmetrical Component Example 1

$$\text{Let } \mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle -120^\circ \\ 10\angle 120^\circ \end{bmatrix} \qquad \text{Then}$$

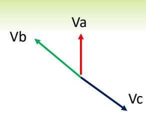
$$\mathbf{I}_s = \mathbf{A}^{-1}\mathbf{I} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 10\angle 0^\circ \\ 10\angle -120^\circ \\ 10\angle 120^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 10\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\text{If } \mathbf{I} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle +120^\circ \\ 10\angle -120^\circ \end{bmatrix} \rightarrow \mathbf{I}_s = \begin{bmatrix} 0 \\ 0 \\ 10\angle 0^\circ \end{bmatrix}$$

14

Symmetrical Component Example 2

$$\text{Let } \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 5\angle 90^\circ \\ 8\angle 150^\circ \\ 8\angle -30^\circ \end{bmatrix}$$



Then

$$\mathbf{V}_s = \mathbf{A}^{-1}\mathbf{V} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 5\angle 90^\circ \\ 8\angle 150^\circ \\ 8\angle -30^\circ \end{bmatrix} = \begin{bmatrix} 1.67\angle 90^\circ \\ 3.29\angle -135^\circ \\ 6.12\angle 68^\circ \end{bmatrix}$$

15

Symmetrical Component Example 3

Let $\mathbf{I}_s = \begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ -10\angle 0^\circ \\ 5\angle 0^\circ \end{bmatrix}$

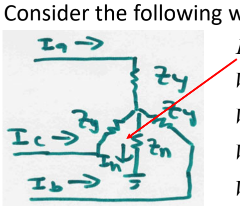
Then

$$\mathbf{I} = \mathbf{A}\mathbf{I}_s = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 10\angle 0^\circ \\ -10\angle 0^\circ \\ 5\angle 0^\circ \end{bmatrix} = \begin{bmatrix} 5.0\angle 0^\circ \\ 18.0\angle 46.1^\circ \\ 18.0\angle -46.1^\circ \end{bmatrix}$$

16

Use of Symmetrical Components

Consider the following wye-connected load:



$$I_n = I_a + I_b + I_c$$
$$V_{ag} = I_a Z_y + I_n Z_n$$
$$V_{ag} = (Z_Y + Z_n) I_a + Z_n I_b + Z_n I_c$$
$$V_{bg} = Z_n I_a + (Z_Y + Z_n) I_b + Z_n I_c$$
$$V_{cg} = Z_n I_a + Z_n I_b + (Z_Y + Z_n) I_c$$

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

17

Use of Symmetrical Components

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I}, \quad \mathbf{V} = \mathbf{A}\,\mathbf{V}_s, \quad \mathbf{I} = \mathbf{A}\,\mathbf{I}_s$$

$$\mathbf{A}\,\mathbf{V}_s = \mathbf{Z}\,\mathbf{A}\,\mathbf{I}_s \quad \rightarrow \quad \mathbf{V}_s = \mathbf{A}^{-1}\,\mathbf{Z}\,\mathbf{A}\,\mathbf{I}_s$$

$$\mathbf{A}^{-1}\,\mathbf{Z}\,\mathbf{A} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} = \mathbf{Z}_S$$

18

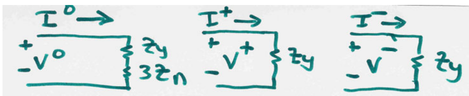
Networks are Now Decoupled

$$\begin{bmatrix} V^0 \\ V^+ \\ V^- \end{bmatrix} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} \leftarrow \mathbf{V_s} = \mathbf{Z_s I_s}$$

Systems are decoupled

$V^0 = (Z_y + 3Z_n)\, I^0 \qquad V^+ = Z_y\, I^+$

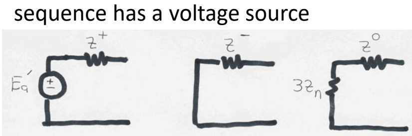
$V^- = Z_y\, I^-$



19

Sequence diagrams for generators

- Key point: generators only produce positive sequence voltages; therefore, only the positive sequence has a voltage source

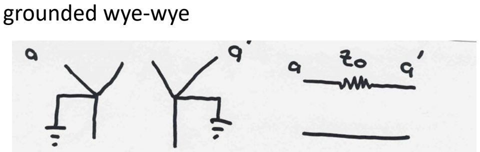


During a fault $Z^+ \approx Z^- \approx X_d''$. The zero sequence impedance is usually substantially smaller. The value of Z_n depends on whether the generator is grounded

20

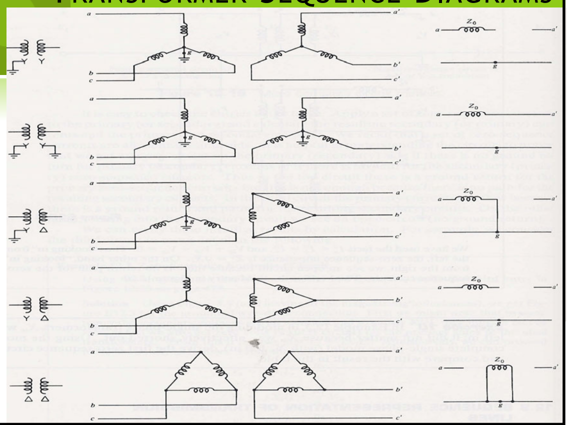
Sequence diagrams for Transformers

- The positive and negative sequence diagrams for transformers are similar to those for transmission lines.
- The zero-sequence network depends upon both how the transformer is grounded and its type of connection. The easiest to understand is a double grounded wye-wye



21

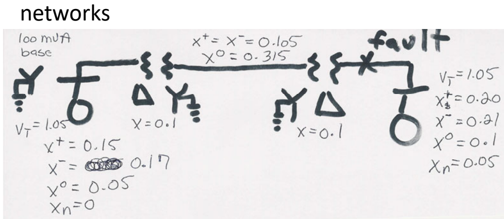
Transformer Sequence Diagrams



22

Unbalanced Fault Analysis

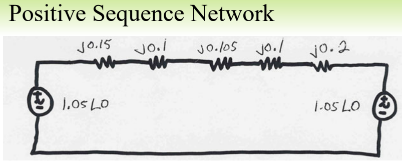
- The first step in the analysis of unbalanced faults is to assemble the three sequence networks. For example, for the earlier single generator, single motor example let's develop the sequence networks



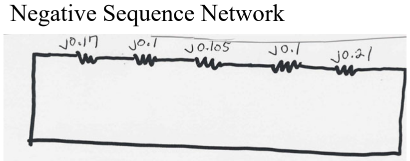
23

Sequence Diagrams for Example

Positive Sequence Network

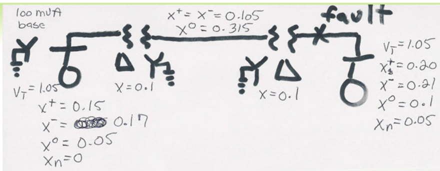


Negative Sequence Network

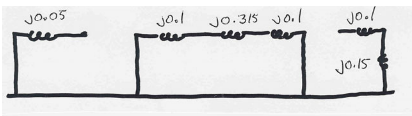


24

Sequence Diagrams for Example

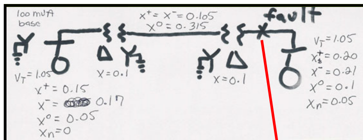


Zero Sequence Network

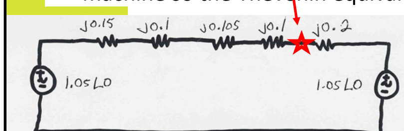


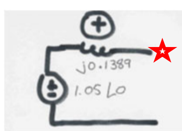
25

Thevenin Equivalents



- To do further analysis we first need to calculate the Thevenin equivalents as seen from the fault location. In this example the fault is at the terminal of the right machine so the Thevenin equivalents are:

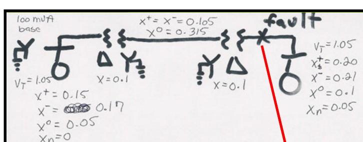




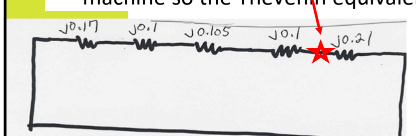
Positive Sequence Network $Z_{th}^{+} = j0.2$ in parallel with j0.455

26

Thevenin Equivalents

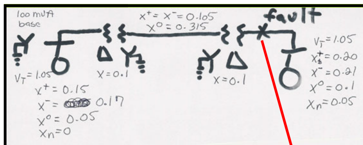


- To do further analysis we first need to calculate the Thevenin equivalents as seen from the fault location. In this example the fault is at the terminal of the right machine so the Thevenin equivalents are:



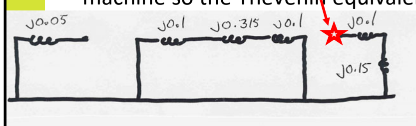
Negative Sequence Network $Z_{th}^{-} = j0.21$ in parallel with j0.475

27



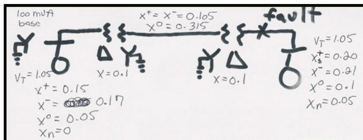
THEVENIN EQUIVALENTS

- To do further analysis we first need to calculate the Thevenin equivalents as seen from the fault location. In this example the fault is at the terminal of the right machine so the Thevenin equivalents are:



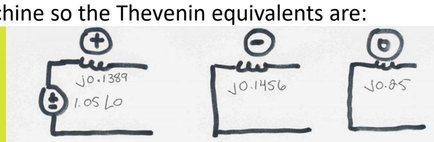
Zero Sequence Network $Z_{th}^0 = j0.1$ and $j0.15$ in series

28



THEVENIN EQUIVALENTS

- To do further analysis we first need to calculate the Thevenin equivalents as seen from the fault location. In this example the fault is at the terminal of the right machine so the Thevenin equivalents are:



$Z_{th}^+ = j0.2$ in parallel with j0.455

$Z_{th}^- = j0.21$ in parallel with j0.475

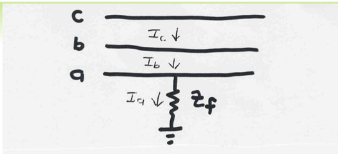
29

SINGLE LINE-TO-GROUND (SLG) FAULTS

- Unbalanced faults unbalance the network, but only at the fault location. This causes a coupling of the sequence networks. How the sequence networks are coupled depends upon the fault type. We'll derive these relationships for several common faults.
- With a SLG fault, only one phase has non-zero fault current -- we'll assume it is phase A.

30

SLG Faults, cont'd



$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} ? \\ 0 \\ 0 \end{bmatrix}$$

Then since

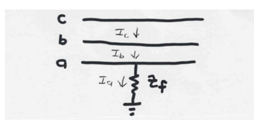
$$\begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a^f \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} I_f^0 = I_f^+ = I_f^- = \frac{1}{3} I_a^f \\ I_a^f = 3 I_f^0 \end{matrix}$$

31

SLG Faults, cont'd

$$V_a^f = Z_f I_a^f = Z_f\left(3 I_f^0\right) = 3 Z_f I_f^0$$

$$\begin{bmatrix} V_a^f \\ V_b^f \\ V_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix}$$

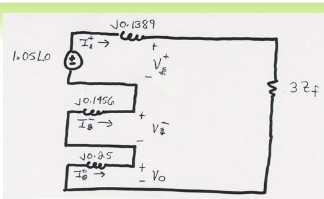


This means $V_a^f = V_f^0 + V_f^+ + V_f^- = 3 Z_f I_f^0$

The only way these two constraints can be satisified is by coupling the sequence networks in series

32

SLG Faults, cont'd



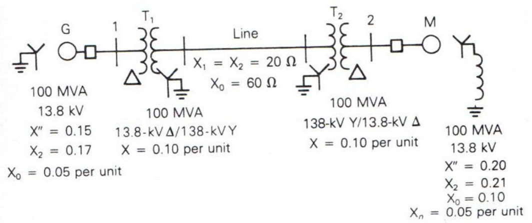
With the sequence networks in series we can solve for the fault currents (assume Z_f=0)

$$I_f^+ = \frac{1.05\angle 0^\circ}{j(0.1389 + 0.1456 + 0.25 + 3Z_f)} = -j1.964 = I_f^- = I_f^0$$

$$\mathbf{I} = \mathbf{A}\,\mathbf{I}_s \rightarrow I_a^f = -j5.8 \ \ \text{(of course, } I_b^f = I_c^f = 0\text{)}$$

33

EXAMPLE 9.3



34